Chapter 19 – Confidence Intervals for Proportions

1. **Margin of error.**

   He believes the true proportion of voters with a certain opinion is within 4% of his estimate, with some degree of confidence, perhaps 95% confidence.

2. **Margin of error.**

   He believes the true percentage of children who are exposed to lead-base paint is within 3% of his estimate, with some degree of confidence, perhaps 95% confidence.

3. **Conditions.**

   a) *Population* – all cars; *sample* – 134 cars actually stopped at the checkpoint; p – proportion of all cars with safety problems; $\hat{p}$ – proportion of cars in the sample that actually have safety problems (10.4%).
   Plausible Independence condition: There is no reason to believe that the safety problems of cars are related to each other.
   Randomization condition: This sample is not random, so hopefully the cars stopped are representative of cars in the area.
   10% condition: The 134 cars stopped represent a small fraction of all cars, certainly less than 10%.
   Success/Failure condition: $n\hat{p} = 14$ and $n\hat{q} = 120$ are both greater than 10, so the sample is large enough.
   A one-proportion z-interval can be created for the proportion of all cars in the area with safety problems.

   b) *Population* – the general public; *sample* – 602 viewers that logged on to the Web site; p –proportion of the general public that support prayer in school; $\hat{p}$ - proportion of viewers that logged on to the Web site and voted that support prayer in schools (81.1%).
   Randomization condition: This sample is not random, but biased by voluntary response. It would be very unwise to attempt to use this sample to infer anything about the opinion of the general public related to school prayer.

   c) *Population* – parents at the school; *sample* – 380 parents who returned surveys; p – proportion of all parents in favor of uniforms; $\hat{p}$ – proportion of those who responded that are in favor of uniforms (60%).
   Randomization condition: This sample is not random, but rather biased by nonresponse. There may be lurking variables that affect the opinions of parents who return surveys (and the children who deliver them!).
   It would be very unwise to attempt to use this sample to infer anything about the opinion of the parents about uniforms.

   d) *Population* – all freshmen enrollees at the college (not just one year); *sample* – 1632 freshmen during the specified year; p – proportion of all students who will graduate on time; $\hat{p}$ – proportion of students from that year who graduate on time (85.05%).
   Plausible independence condition: It is reasonable to think that the abilities of students to graduate on time are mutually independent.

Randomization condition: This sample is not random, but this year's freshmen class is probably representative of freshman classes in other years.
10% condition: The 1632 students in that years freshmen class represent less than 10% of all possible students.
Success/Failure condition: $n\hat{p} = 1388$ and $n\hat{q} = 244$ are both greater than 10, so the sample is large enough.
A one-proportion *z*-interval can be created for the proportion of freshmen that graduate on time from this college.

4. More conditions.

a) *Population* – all customers who recently bought new cars; *sample* – 167 people surveyed about their experience; p – proportion of all new car buyers who are dissatisfied with the salesperson; $\hat{p}$ – proportion of new car buyers surveyed who are dissatisfied with the salesperson (3%).
Success/Failure condition: $n\hat{p} = 167(0.03) = 5$ and $n\hat{q} = 162$. Since only 5 people were dissatisfied, the sample is **not** large enough to use a confidence interval to estimate the proportion of dissatisfied car buyers.

b) *Population* – all college students; *sample* – 2883 who were asked about their cell phones at the football stadium; p – proportion of all college students with cell phones; $\hat{p}$ – proportion of college students at the football stadium with cell phones (8.4%).
Plausible independence condition: Whether or not a student has a cell phone shouldn't affect the probability that another does.
Randomization condition: This sample is not random. The best we can hope for is that the students at the football stadium are representative of all college students.
10% condition: The 2883 students at the football stadium represent less than 10% of all college students.
Success/Failure condition: $n\hat{p} = 243$ and $n\hat{q} = 2640$ are both greater than 10, so the sample is large enough.
Extreme caution should be used when using a one-proportion *z*-interval to estimate the proportion of college students with cell phones. The students at the football stadium may not be representative of all students.

c) *Population* – potato plants in the U.S.; *sample* – 240 potato plants in a field in Maine; p – proportion of all potato plants in the U.S. that show signs of blight; $\hat{p}$ – proportion of potato plants in the sample that show signs of blight (2.9%).
Plausible independence condition: It is **not** reasonable to think that signs of blight are independent. Blight is a contagious disease!
Randomization condition: Although potato plants are randomly selected from the field in Maine, it doesn't seem reasonable to assume that these potato plants are representative of all potato plants in the U.S.
Success/Failure condition: $n\hat{p} = 7$ and $n\hat{q} = 233$. There are only 7 (less than 10!) plants with signs of blight. The sample is not large enough.
Three conditions are not met! Don't use a confidence interval to attempt to estimate the percentage of potato plants in the U.S. that show signs of blight.

d) *Population* – all employees at the company; *sample* – all employees during the specified year; p – proportion of all employees who will have an injury on the job in a year; $\hat{p}$ – proportion of employees who had an injury on the job during the specified year.
Plausible independence condition: It is reasonable to think that the injuries are mutually independent.
Randomization condition: This sample is not random, but this year's employees are probably representative of employees in other years, with regards to injury on the job.
10% condition: The 309 employees represent less than 10% of all possible employees over many years.
Success/Failure condition: $n\hat{p} = 12$ and $n\hat{q} = 297$ are both greater than 10, so the sample is large enough.
A one-proportion z-interval can be created for the proportion of employees who are expected to suffer an injury on the job in future years, provided that this year is representative of future years.

5. Conclusions.

a) Not correct. This statement implies certainty. There is no level of confidence in the statement.

b) Not correct. Different samples will give different results. Many fewer than 95% of samples are expected to have *exactly* 88% on-time orders.

c) Not correct. A confidence interval should say something about the unknown population proportion, not the sample proportion in different samples.

d) Not correct. We *know* that 88% of the orders arrived on time. There is no need to make an interval for the sample proportion.

e) Not correct. The interval should be about the proportion of on-time orders, not the days.

6. More conclusions.

a) Not correct. This statement implies certainty. There is no level of confidence in the statement.

b) Not correct. We *know* that 56% of the spins in this experiment landed heads. There is no need to make an interval for the sample proportion.

c) Correct.

d) Not correct. The interval should be about the proportion of heads, not the spins.

e) Not correct. The interval should be about the proportion of heads, not the percentage of euros.

7. Confidence intervals.

a) False. For a given sample size, higher confidence means a *larger* margin of error.

b) True. Larger samples lead to smaller standard errors, which lead to smaller margins of error.

c) True. Larger samples are less variable, which makes us more confident that a given confidence interval succeeds in catching the population proportion.

d) False. The margin of error decreases as the square root of the sample size increases. Halving the margin of error requires a sample four times as large as the original.

8. Confidence intervals, again.

a) True. The smaller the margin of error is, the less confidence we have in the ability of our interval to catch the population proportion.

b) True. Larger samples are less variable, which translates to a smaller margin of error. We can be more precise at the same level of confidence.

c) True. Smaller samples are more variable, leading us to be less confident in the ability of our interval to catch the true population proportion.

d) True. The margin of error decreases as the square root of the sample size increases.

9. Cars.

We are 90% confident that between 29.9% and 47.0% of cars are made in Japan.

10. Parole.

We are 95% confident that between 56.1% and 62.5% of paroles are granted by the Nebraska Board of Parole.

11. Contaminated chicken.

a) $\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.83) \pm 1.960\sqrt{\frac{(0.83)(0.17)}{525}} = (0.798, 0.862)$

b) We are 95% confident that between 80% and 86% of all broiler chicken sold in U.S. food stores is infected with *campylobacter*.

c) The size of the population is irrelevant. If *Consumer Reports* had a random sample, 95% of intervals generated by studies like this are expected to capture the true contamination level.

12. Contaminated chicken, second course.

a) **Independence assumption:** It is very important that the researchers kept the chicken samples separated. Otherwise, *salmonella* could be transmitted from sample to sample.
Randomization condition: It's not clear how the sample was chosen. We will assume the sample from 23 states is at least representative of all broiler chicken sold.
10% condition: 525 is far less than 10% of all broiler chickens.
Success/Failure condition: $n\hat{p} = 79$ and $n\hat{q} = 446$ are both greater than 10, so the sample is large enough.

b) Since the conditions are met, we can use a one-proportion z-interval to estimate the percentage of broiler chicken packages that are infected with *salmonella*.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.15) \pm 1.960\sqrt{\frac{(0.15)(0.85)}{525}} = (0.12, 0.18)$$

c) We are 95% confident that between 12% and 18% of all broiler chicken sold in U.S. food stores is infected with *salmonella*.

13. Baseball fans.

a) $ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \times \sqrt{\frac{(0.37)(0.63)}{1006}} \approx 0.025$

b) We're 90% confident that this poll's estimate is within 2.5% of the true proportion of people who are baseball fans.

c) The margin of error for 99% confidence would be larger. To be more certain, we must be less precise.

d) $ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.576 \times \sqrt{\frac{(0.37)(0.63)}{1006}} \approx 0.039$

e) Smaller margins of error involve less confidence. The narrower the confidence interval, the less likely we are to believe that we have succeeded in capturing the true proportion of people who are baseball fans.

f) These data provide no evidence of a change. With a margin of error of 2.5% for the 95% confidence interval, 37% is certainly still a plausible value for the proportion of people who are baseball fans.

14. Cloning 2007.

a) $ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.960 \times \sqrt{\frac{(0.11)(0.89)}{1003}} \approx 1.9\%$

b) The pollsters are 95% confident that the true proportion of adults who approve of attempts to clone a human is within 1.9% of the estimated 11%.

c) A 90% confidence interval results in a smaller margin of error. If confidence is decreased, a smaller interval is constructed.

d) $ME = z^* \times SE(\hat{p}) = z^* \times \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \times \sqrt{\frac{(0.11)(0.89)}{1003}} \approx 1.6\%$

e) Smaller samples generally produce larger intervals. Smaller samples are more variable, which increases the margin of error.

15. Contributions please.

a) **Randomization condition:** Letters were sent to a random sample of 100,000 potential donors.
10% condition: We assume that the potential donor list has more than 1,000,000 names.
Success/Failure condition: $n\hat{p} = 4{,}781$ and $n\hat{q} = 95{,}219$ are both much greater than 10, so the sample is large enough.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{4{,}781}{100{,}000}\right) \pm 1.960 \sqrt{\frac{\left(\frac{4{,}781}{100{,}000}\right)\left(\frac{95{,}219}{100{,}000}\right)}{100{,}000}} = (0.0465, 0.0491)$$

We are 95% confident that the between 4.65% and 4.91% of potential donors would donate.

b) The confidence interval gives the set of plausible values with 95% confidence. Since 5% is outside the interval, it seems to be a bit optimistic.

16. Take the offer.

a) **Randomization condition:** Offers were sent to a random sample of 50,000 cardholders.
10% condition: We assume that there are more than 500,000 cardholders.
Success/Failure condition: $n\hat{p} = 1{,}184$ and $n\hat{q} = 48{,}816$ are both much greater than 10, so the sample is large enough.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{1{,}184}{50{,}000}\right) \pm 1.960 \sqrt{\frac{\left(\frac{1{,}184}{50{,}000}\right)\left(\frac{48{,}816}{50{,}000}\right)}{50{,}000}} = (0.0223, 0.0250)$$

We are 95% confident that the between 2.23% and 2.5% of all cardholders would register for double miles.

b) The confidence interval gives the set of plausible values with 95% confidence. Since 2% is outside the interval, there is evidence that the true proportion is above 2%. The campaign should be worth the expense.

17. Teenage drivers.

a) **Independence assumption:** There is no reason to believe that accidents selected at random would be related to one another.
Randomization condition: The insurance company randomly selected 582 accidents.
10% condition: 582 accidents represent less than 10% of all accidents.
Success/Failure condition: $n\hat{p} = 91$ and $n\hat{q} = 491$ are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the percentage of accidents involving teenagers.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{91}{582}\right) \pm 1.960 \sqrt{\frac{\left(\frac{91}{582}\right)\left(\frac{491}{582}\right)}{582}} = (12.7\%, 18.6\%)$$

b) We are 95% confident that between 12.7% and 18.6% of all accidents involve teenagers.

c) About 95% of random samples of size 582 will produce intervals that contain the true proportion of accidents involving teenagers.

d) Our confidence interval contradicts the assertion of the politician. The figure quoted by the politician, 1 out of every 5, or 20%, is outside the interval.

18. Junk mail.

a) **Independence assumption:** There is no reason to believe that one randomly selected person's response will affect another's.
Randomization condition: The company randomly selected 1000 recipients.
10% condition: 1000 recipients is less than 10% of the population of 200,000 people.
Success/Failure condition: $n\hat{p} = 123$ and $n\hat{q} = 877$ are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the percentage of people who will respond to the new flyer.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{123}{1000}\right) \pm 1.645 \sqrt{\frac{\left(\frac{123}{1000}\right)\left(\frac{877}{1000}\right)}{1000}} = (10.6\%, 14.0\%)$$

b) We are 90% confident that between 10.6% and 14.0% of people will respond to the new flyer.

c) About 90% of random samples of size 1000 will produce intervals that contain the true proportion of people who will respond to the new flyer.

d) Our confidence interval suggests that the company should do the mass mailing. The entire interval is well above the cutoff of 5%.

19. Safe food.

The grocer can conclude nothing about the opinions of all his customers from this survey. Those customers who bothered to fill out the survey represent a voluntary response sample, consisting of people who felt strongly one way or another about irradiated food. The random condition was not met.

20. Local news.

The city council can conclude nothing about general public support for the mayor's initiative. Those who showed up for the meeting are probably a biased group. In addition, a show of hands vote may influence people, affecting the independence of the votes.

21. Death penalty, again.

a) There may be response bias based on the wording of the question.

b) $$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.57) \pm 1.960 \sqrt{\frac{(0.57)(0.43)}{1020}} = (54\%, 60\%)$$

c) The margin of error based on the pooled sample is smaller, since the sample size is larger.

22. Gambling.

a) The interval based on the survey conducted by the college Statistics class will have the larger margin of error, since the sample size is smaller.

b) **Independence assumption:** There is no reason to believe that one randomly selected voter's response will influence another.
Randomization condition: Both samples were random.
10% condition: Both samples are probably less than 10% of the city's voters, provided the city has more than 12,000 voters.
Success/Failure condition:
For the newspaper, $n_1\hat{p}_1 = (1200)(0.53) = 636$ and $n_1\hat{q}_1 = (1200)(0.47) = 564$
For the Statistics class, $n_2\hat{p}_2 = (450)(0.54) = 243$ and $n_2\hat{q}_2 = (450)(0.46) = 207$
All the expected successes and failures are greater than 10, so the samples are large enough.

Since the conditions are met, we can use one-proportion z-intervals to estimate the proportion of the city's voters that support the gambling initiative.

Newspaper poll: $\hat{p}_1 \pm z^* \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}} = (0.53) \pm 1.960\sqrt{\frac{(0.53)(0.47)}{1200}} = (50.2\%, 55.8\%)$

Statistics class poll: $\hat{p}_2 \pm z^* \sqrt{\frac{\hat{p}_2\hat{q}_2}{n_2}} = (0.54) \pm 1.960\sqrt{\frac{(0.54)(0.46)}{450}} = (49.4\%, 58.6\%)$

c) The Statistics class should conclude that the outcome is too close to call, because 50% is in their interval.

23. Rickets.

a) **Independence assumption:** It is reasonable to think that the randomly selected children are mutually independent in regards to vitamin D deficiency.
Randomization condition: The 2,700 children were chosen at random.
10% condition: 2,700 children are less than 10% of all English children.
Success/Failure condition: $n\hat{p}$ = (2,700)(0.20) = 540 and $n\hat{q}$ = (2,700)(0.80) = 2160 are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the proportion of the English children with vitamin D deficiency.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = (0.20) \pm 2.326\sqrt{\frac{(0.20)(0.80)}{2700}} = (18.2\%, 21.8\%)$$

b) We are 98% confident that between 18.2% and 21.8% of English children are deficient in vitamin D.

c) About 98% of random samples of size 2,700 will produce confidence intervals that contain the true proportion of English children that are deficient in vitamin D.

24. Pregnancy.

a) **Independence assumption:** There is no reason to believe that one woman's ability to conceive would affect others.
Randomization condition: These women are not chosen at random. Assume that they are representative of all women under 40 that had previously been unable to conceive.
10% condition: 207 women is less than 10% of all such women.
Success/Failure condition: $n\hat{p}$ = 49 and $n\hat{q}$ = 158 are both greater than 10, so the sample is large enough.

Since the conditions are met, we can use a one-proportion z-interval to estimate the proportion of the births to women at the clinic.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{49}{207}\right) \pm 1.645\sqrt{\frac{\left(\frac{49}{207}\right)\left(\frac{158}{207}\right)}{207}} = (18.8\%, 28.5\%)$$

b) We are 90% confident that between 18.8% and 28.5% of women under 40 who are treated at this clinic will give birth.

c) About 90% of random samples of size 207 will produce confidence intervals that contain the true proportion of women under 40 who are treated at this clinic that will give birth.

d) These data do not refute the clinics claim of a 25% success rate, since 25% is in the interval.

25. Payments.

a) The confidence interval will be wider. The sample size is probably about one-fourth of the sample size of for all adults, so we would expect the confidence interval to be about twice as wide.

b) The second poll's margin of error should be smaller. The second poll used a slightly larger sample.

26. Back to campus.

a) The confidence interval for the retention rate in private colleges will be narrower than the confidence interval for the retention rate in public colleges, since it is based on a larger sample.

b) Since the overall sample size is larger, the margin of error in retention rate is expected to be smaller.

27. Deer ticks.

a) **Independence assumption:** Deer ticks are parasites. A deer carrying the parasite may spread it to others. Ticks may not be distributed evenly throughout the population.
Randomization condition: The sample is not random and may not represent all deer.
10% condition: 153 deer are less than 10% of all deer.
Success/Failure condition: $n\hat{p} = 32$ and $n\hat{q} = 121$ are both greater than 10, so the sample is large enough.

The conditions are not satisfied, so we should use caution when a one-proportion z-interval is used to estimate the proportion of deer carrying ticks.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = \left(\frac{32}{153}\right) \pm 1.645 \sqrt{\frac{\left(\frac{32}{153}\right)\left(\frac{121}{153}\right)}{153}} = (15.5\%, 26.3\%)$$

We are 90% confident that between 15.5% and 26.3% of deer have ticks.

b) In order to cut the margin of error in half, they must sample 4 times as many deer. 4(153) = 612 deer.

c) The incidence of deer ticks is not plausibly independent, and the sample may not be representative of all deer, since females and young deer are usually not hunted.

28. Pregnancy II.

a) In order to cut the margin of error in half, they must use 4 times as many patient results. 4(207) = 828.

b) A sample this large may be more than 10% of the population of all potential patients.

29. Graduation.

a)

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.06 = 1.645\sqrt{\frac{(0.25)(0.75)}{n}}$$

$$n = \frac{(1.645)^2(0.25)(0.75)}{(0.06)^2}$$

$$n \approx 141 \text{ people}$$

In order to estimate the proportion of non-graduates in the 25-to 30-year-old age group to within 6% with 90% confidence, we would need a sample of at least 141 people. All decimals in the final answer must be rounded up, to the next person.
(For a more cautious answer, let $\hat{p} = \hat{q} = 0.5$. This method results in a required sample of 188 people.)

b)

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.04 = 1.645\sqrt{\frac{(0.25)(0.75)}{n}}$$

$$n = \frac{(1.645)^2(0.25)(0.75)}{(0.04)^2}$$

$$n \approx 318 \text{ people}$$

In order to estimate the proportion of non-graduates in the 25-to 30-year-old age group to within 4% with 90% confidence, we would need a sample of at least 318 people. All decimals in the final answer must be rounded up, to the next person.
(For a more cautious answer, let $\hat{p} = \hat{q} = 0.5$. This method results in a required sample of 423 people.)
Alternatively, the margin of error is now 2/3 of the original, so the sample size must be increased by a factor of 9/4. $141(9/4) \approx 318$ people.

c)

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.03 = 1.645\sqrt{\frac{(0.25)(0.75)}{n}}$$

$$n = \frac{(1.645)^2(0.25)(0.75)}{(0.03)^2}$$

$$n \approx 564 \text{ people}$$

In order estimate the proportion of non-graduates in the 25-to 30-year-old age group to within 3% with 90% confidence, we would need a sample of at least 564 people. All decimals in the final answer must be rounded up, to the next person.
(For a more cautious answer, let $\hat{p} = \hat{q} = 0.5$. This method results in a required sample of 752 people.)

Alternatively, the margin of error is now half that of the original, so the sample size must be increased by a factor of 4. $141(4) \approx 564$ people.

30. Hiring.

a)

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.05 = 2.326\sqrt{\frac{(0.5)(0.5)}{n}}$$

$$n = \frac{(2.326)^2(0.5)(0.5)}{(0.05)^2}$$

$$n \approx 542 \text{ businesses}$$

In order to estimate the percentage of businesses planning to hire additional employees within the next 60 days to within 5% with 98% confidence, we would need a sample of at least 542 businesses. All decimals in the final answer must be rounded up, to the next business.

b)

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.03 = 2.326\sqrt{\frac{(0.5)(0.5)}{n}}$$

$$n = \frac{(2.326)^2(0.5)(0.5)}{(0.03)^2}$$

$$n \approx 1503 \text{ businesses}$$

In order to estimate the percentage of businesses planning to hire additional employees within the next 60 days to within 3% with 98% confidence, we would need a sample of at least 1503 businesses. All decimals in the final answer must be rounded up, to the next business.

(Alternatively, the margin of error is being decreased to 3/5 of its original size, so the sample size must increase by a factor of 25/9. 542(25/9) $\approx$ 1506 businesses. A bit off, because 542 was rounded, but close enough!

c)

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.01 = 2.326\sqrt{\frac{(0.5)(0.5)}{n}}$$

$$n = \frac{(2.326)^2(0.5)(0.5)}{(0.01)^2}$$

$$n \approx 13{,}526 \text{ businesses}$$

In order to estimate the percentage of businesses planning to hire additional employees within the next 60 days to within 1%with 98% confidence, we would need a sample of at least 13,526 businesses.

(Alternatively, the margin of error has been decreased to 1/5 of its original size, so a sample 25 times as large would be needed. 25(542) = 13,550. Close enough!

It would probably be very expensive and time consuming to sample that many businesses.

31. Graduation, again.

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.02 = 1.960\sqrt{\frac{(0.25)(0.75)}{n}}$$

$$n = \frac{(1.960)^2(0.25)(0.75)}{(0.02)^2}$$

$$n \approx 1{,}801 \text{ people}$$

In order to estimate the proportion of non-graduates in the 25-to 30-year-old age group to within 2% with 95% confidence, we would need a sample of at least 1,801 people. All decimals in the final answer must be rounded up, to the next person.
(For a more cautious answer, let $\hat{p} = \hat{q} = 0.5$. This method results in a required sample of 2,401 people.)

32. Better hiring info.

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.04 = 2.576\sqrt{\frac{(0.5)(0.5)}{n}}$$

$$n = \frac{(2.576)^2(0.5)(0.5)}{(0.04)^2}$$

$$n \approx 1{,}037 \text{ businesses}$$

In order to estimate the percentage of businesses planning to hire additional employees within the next 60 days to within 4% with 99% confidence, we would need a sample of at least 1,037 businesses. All decimals in the final answer must be rounded up, to the next business.

33. Pilot study.

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.03 = 1.645\sqrt{\frac{(0.15)(0.85)}{n}}$$

$$n = \frac{(1.645)^2(0.15)(0.85)}{(0.03)^2}$$

$$n \approx 384 \text{ cars}$$

Use $\hat{p} = \frac{9}{60} = 0.15$ from the pilot study as an estimate.

In order to estimate the percentage of cars with faulty emissions systems to within 3% with 90% confidence, the state's environmental agency will need a sample of at least 384 cars. All decimals in the final answer must be rounded up, to the next car.

34. Another pilot study.

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.04 = 2.326\sqrt{\frac{(0.22)(0.78)}{n}}$$

$$n = \frac{(2.326)^2(0.22)(0.78)}{(0.04)^2}$$

$$n \approx 581 \text{ adults}$$

Use $\hat{p} = 0.22$ from the pilot study as an estimate.

In order to estimate the percentage of adults with higher than normal levels of glucose in their blood to within 4% with 98% confidence, the researchers will need a sample of at least 581 adults. All decimals in the final answer must be rounded up, to the next adult.

35. Approval rating.

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.025 = z^*\sqrt{\frac{(0.65)(0.35)}{972}}$$

$$z^* = \frac{0.025}{\sqrt{\frac{(0.65)(0.35)}{972}}}$$

$$z^* \approx 1.634$$

Since $z^* \approx 1.634$, which is close to 1.645, the pollsters were probably using 90% confidence. The slight difference in the z^* values is due to rounding of the governor's approval rating.

36. Amendment.

a) This poll is inconclusive because the confidence interval, 52% ± 3% contains 50%. The true proportion of voters in favor of the constitutional amendment is estimated to be between 49% (minority) to 55% (majority). We can't be sure whether or not the majority of voters support the amendment or not.

b)

$$ME = z^*\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.03 = z^*\sqrt{\frac{(0.52)(0.48)}{1505}}$$

$$z^* = \frac{0.03}{\sqrt{\frac{(0.52)(0.48)}{1505}}}$$

$$z^* \approx 2.3295$$

Since $z^* \approx 2.3295$, which is close to 2.326, the pollsters were probably using 98% confidence. The slight difference in the z^* values is due to rounding of the amendment's approval rating.